SEQUENCES AND SERIES PG 114-127

First let's summarise the objectives of this section. By the end you should be able to:

- Know how a sequence is constructed from a formula or inductive definition
- Be familiar with triangle, factorial, Pascal and arithmetic sequences
- Find the sum of an arithmetic series

1. Warming Up

You should all be familiar with simple sequences like those given on page 114 in section 8.1 and could easily complete the next item in the series.

The trick becomes how to write these as formulae. For each series, there is a formula but equally important is the first term. To fully define the series it is necessary to have both the formula and the first term – together known as an inductive definition.

Watch these links before progressing

http://www.examsolutions.co.uk/maths-tutorials/Core-maths/sequences/recurrence-relations/tutorial.php

http://www.examsolutions.co.uk/maths-tutorials/Core-maths/arithmetic_progression/tutorial.php

Work through section 8.1 and then....

Activity:

To test your knowledge and have some fun, work through:

- Exercise A – Question 1 - 4

Mark your answers. How did you do?

2. *The triangle number sequence, factorial sequence and Pascal sequences*

Triangle number sequence

Follow through the logic on page 116 and diagrams 8.1 – 8.3 to arrive at the formula for all natural numbers:

The sum of all natural numbers from 1 to r ....

= ½ r (r + 1)

Now we can put this into algebraic form. Follow the steps on page 117.

Factorial number sequence

If instead of adding, you go from one term to the next by multiplying, this is the factorial sequence. Read through section 8.3. This sequence is very important and we will return to it in later work.

Pascal sequences

This is another very important type of sequences. Work through section 8.4.

Note that the general definition for a Pascal sequence using the terms *n* and *r*

Activity:

Spend some time and work through the questions in Exercise 8B.

These examples should be fun but will stretch your mind a bit !

Check your answers at the back.

3. Arithmetic sequences

An arithmetic sequence or progression is a sequence where the numbers go up or down by a constant amount. This can be written as

Simply put:

$$u_1 = a, \text{ and } \quad u_{r+1} = u_r + d$$

Such a sequence is easy when there are only a few terms but this is rarely the case! Work through Example 8.5 on page 120 and 121 which shows how to approach this and how to derive a formula for an arithmetic series. You will notice a similar logic to that used earlier in this chapter.

You should now be able to generate the nth term of an arithmetic sequence a+ (n – 1)d

Now let's look at deriving a formula to give the sum of the arithmetic series. Two methods are presented on page 122 and you should work through and be comfortable with both. Of course in the end they give the same formula !

Watch this link and then move on http://www.examsolutions.co.uk/maths-tutorials/Core-maths/arithmetic_progression/tutorial.php

An arithmetic series with

n terms, first term = a and a common difference = d

- the last term is given by, $l = a + (n - 1)d$
- the sum is given by

  S = ½ $n(a+1)$ = ½ $n(2a+(n-1)\,d)$

You should now be able to use the formula for sum to n terms to sum up the terms.

Work through the examples 8.5.2 and 8.5.3 to test this.

Activity:

Spend some time and work through the questions in Exercise 8C.

Check your answers at the back.

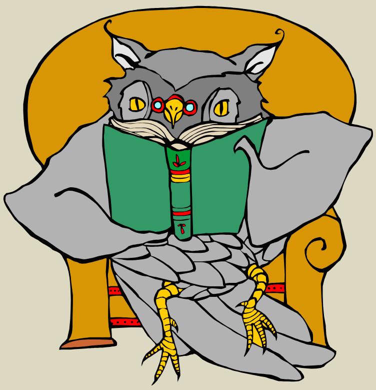
The exercises given in Miscellaneous Exercise 8 build upon the knowledge learnt in this chapter and will stretch your mind. Many of them are past exam questions from other Examination Boards.

It is therefore in your best interest to work through these!

Activity

Work through Miscellaneous Exercise 8 very carefully.

Check your answers and re-work questions where you struggled.



THE BINOMIAL SERIES pg 128-137

First let's summarise the objectives of this section. By the end you should be able to:

- Use Pascal's triangle to find expansion of (x + y)n when n small
- Calculate the coefficients in the above expansion when n is large
- Be able to use notation

1. *Expanding $(x + y)^n$*

The binomial theorem allows us to expand terms of the form $(x + y)^n$ quickly and easily.

On page 128 work through the examples in 9.1 for $(x + y)^{2,}$ $(x + y)^3$ and $(x + y)^4$ and the summary of the results from $(x + y)^1$ to $(x + y)^4$

You should study the expansions carefully. You will notice how

- The power of x starts to the left with x^n and successively reduces by 1
- The powers of y increase by 1 up to y^n
- The coefficients form the pattern of Pascal's triangle.

Read the short section at the top of page 129 on building up Pascal's triangle and then work through examples 9.1.1. to 9.1.5. The last example is rather tricky so make sure you spend enough time working through it, until you are entirely happy.

Activity

Work through Exercise 9A, questions 1 to 11

Take your time, work carefully and make sure you fully understand

2. *The binomial theorem*

The formula above works well for small values of *n* but would take far too long for large values. What is needed is a formula for n and r for the coefficient $x^{n-r} y^{r}$ in the expansion $(x+y)^n$…

Looking back to the previous chapter on Pascal's sequence,

Revision !

= 1 and = ——

Where r = 0,2,3,4 …….

Look on page 131 for the application of this to Pascal's triangle, which enables the derivation of a neater form of the expansion of $(x+y)^n$

The binomial theorem states that if *n* is a natural number….

$$(x+y)^n = \quad x^n + \quad x^{n-1}y + \quad x^{n-1}y^2 + \ldots + \quad y^n$$

Now work through the derivation on pages 131 and 132 of the formula to calculate the coefficients. The end result is

The binomial coefficients are given by…

= ———————— or = ————

Many calculators will give you values of using a $_nC_r$ but check your particular calculators first.

Using these formulae, work through examples 9.2.1 and 9.2.2. You are advised to try working through the proof in example 9.2.1.

Activity

To finish this section….

- Work through Exercise 9B, questions 1 to 13
- Work through Miscellaneous Exercise 9, of which there are 33 questions in total.

Take your time, work carefully and make sure you fully understand.

The questions in the Miscellaneous Exercise will become increasingly difficult but these latter questions provide excellent exam practice.

GEOMETRIC SEQUENCES pg210-223

First let's summarise the objectives of this section. By the end you should be able to:

- Recognise geometric sequences and do calculations on them
- Know and find the sum of a geometric series, including to infinity S_∞.
- Know the condition for a geometric series to converge and find its limiting sum.

1. Geometric sequences – introduction

In the last section, you looked at arithmetic series, where each progresses by adding a constant. In geometric sequences each term is multiplied by a constant.

A geometric sequence is defined by….

$u_1 = a$ and $u_{i+1} = ru_i$

r is the common ratio of the sequence

Note : r ≠ 0 and ≠ 1

See if you can derive the formula for the *i*th term? Compare it to the book on page 210.

The *i*th term is given by $u_i = r^{i-1} \times u_i$ which gives $u_i = ar^{i-1}$ Did you get it ?

Take a look at example 14.1.1 – this gives you a feel for the numbers we will be dealing with.

2. Summing geometric series

The method used to find the sum of an arithmetic series cannot be applied for geometric series and so requires a different approach.

Follow the simple example on page 211 and then work through the proof….

$S = a + ar + ar^2 + ar^3 + ar^4 + \ldots\ldots\ldots ar^{n-2} + ar^{n-1}$

Multiply by r gives

$Sr = ar + ar^2 + ar^3 + ar^4 + \ldots\ldots\ldots ar^{n-1} + ar^n$

The common terms highlighted above are $ar + ar^2 + ar^3 + ar^4 + \ldots\ldots\ldots ar^{n-2} + ar^{n-1}$

which gives

$S - a = Sr - ar^n$

And then

$S(1 - r) = a(1 - r^n)$ or $S = \frac{-}{\quad}$

Note that the formula is valid for both positive and negative values of r

Have a look at the Exams Solutions link

http://www.examsolutions.co.uk/A-Level-maths-tutorials/Edexcel/C2/geometric-progressions/sum-proof/tutorial.php

and then

Work through examples 14.2.1 and 4.2.1. Make sure that you can follow and understand this before moving on.

Activity

Work through Exercise 14A – questions 1 to 12.

The questions start off straightforward but get progressively harder. Make sure you attempt them all

Mark you work using the answers and re-work any questions where you have struggled.

3. *Convergence sequences*

Section 14.3 on page 215 introduces the idea of sum sequences.

For a give sequence u_i, the sum sequences $S_i = u_i + \ldots + u_i$ is defined by

$S_1 = u_1$ and $S_i = S_i + u_{i+1}$

Four geometric sequences are presented on page 215, each of which show different characteristics. Two tend to a limit, the other two do not. Work through this section including examples 14.3.1 and 14.3.2.

We see that if < 1 then the sum of the geometric series tends to infinity. The infinite geometric series in then said to be convergent.

If < 1, the sum of the geometric series with first tem a and common ratio r tends to the limit S_∞ = —— as the number of terms tends to infinity.

The infinite geometric series is convergent

S_∞ is called the sum to infinity of the series

Watch the link

http://www.examsolutions.co.uk/maths-tutorials/Core-maths/series/geometric-progressions/sum-to-infinity/tutorial.php

Work through the examples 14.3.1 and 14.3.2.

Activity
Attempt Exercise 14B – questions 1 to 11.

Check your answers and then re-work any you found difficult or that were incorrect.

4. Exponential growth and decay

The examples in 14.4.1 and 14.4.2 show some ever day applications of geometric sequences. 14.4.3 is more theoretical.

Activity

Attempt Exercise 14C – questions 1 to 11.

Check your answers and then re-work any you found difficult or that were incorrect.

The exercises given in Miscellaneous Exercise 14 build upon the knowledge learnt in this chapter and will stretch your mind. Many of them are past exam questions from other Examination Boards.

It is therefore definitely in your best interest to work through these!